b-stable set interdiction on bipartite graphs

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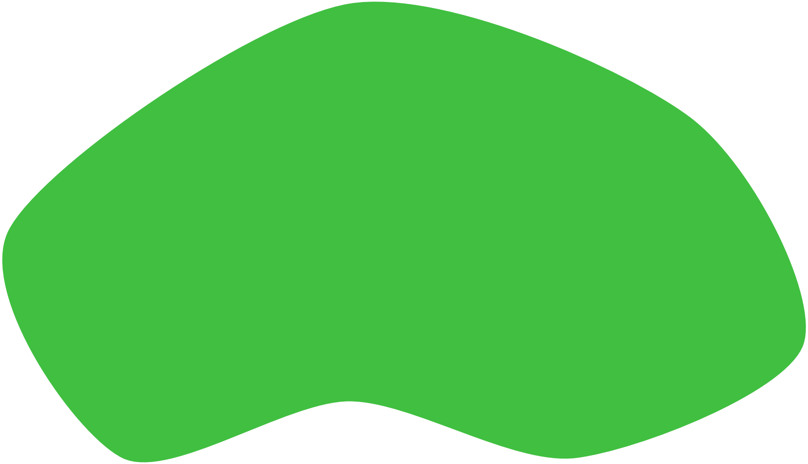
Forests and stable sets



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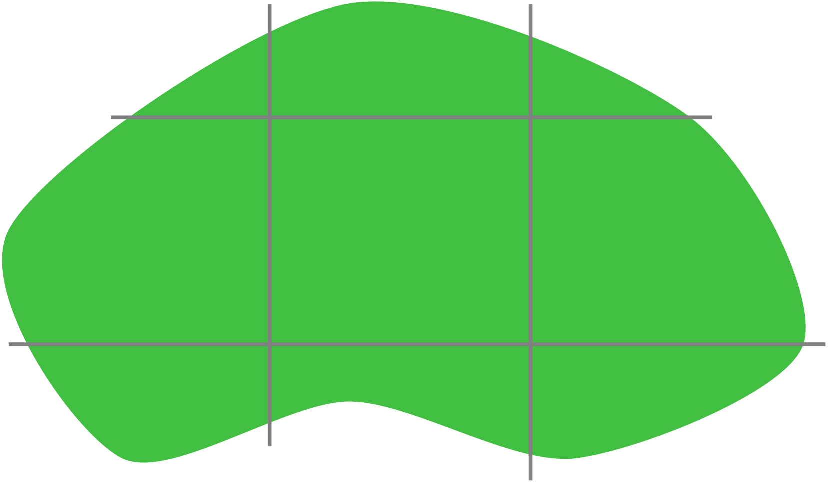

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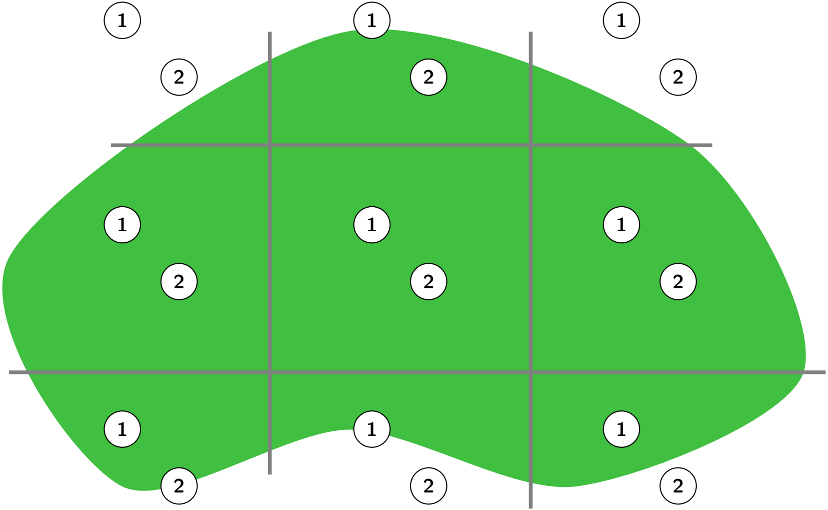

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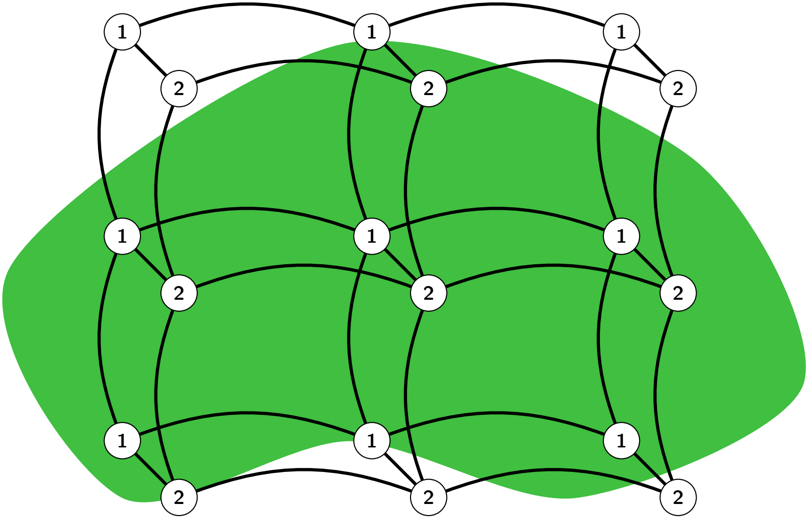

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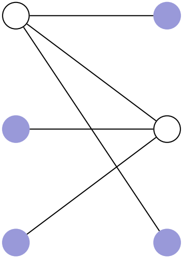
Interdiction overview

Goal Inhibit solutions to an optimization problem by limiting the feasible set

Example Forbid vertices to minimize the maximum stable set



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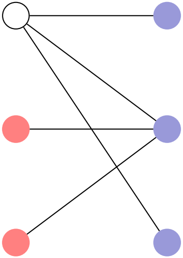

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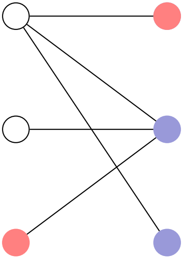


Our results

This talk

Polynomial-time $(1 + \epsilon)$-approximation algorithm for b-stable set interdiction on bipartite graphs

More generally

2-approximation

or

super-optimal with cost $\leq 2B$

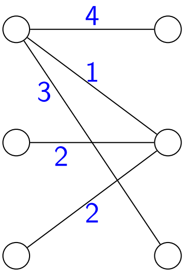
and improvements for some problems

Extending techniques of

Burch, Carr, Krumke, Marathe, Phillips, & Sundberg. "A decomposition based pseudoapproximation algorithm for network flow inhibition." 2003.

b-stable set interdiction in bipartite graphs

$$\begin{array}{lcll} G & = & (V = I \cup J, E) & \text{bipartite graph} \\ b & \in & \mathbb{N}^E & \text{bounds} \end{array}$$



$x \in \mathbb{N}^V$ is b-stable if

$$x(u) + x(v) \leq b_{uv}, \text{ for all } uv \in E.$$

b-stable set interdiction in bipartite graphs

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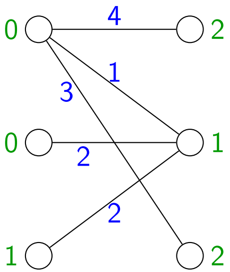

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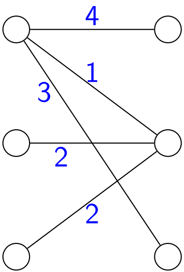

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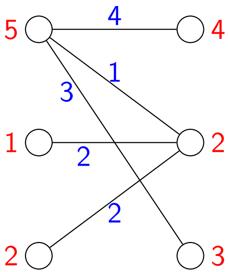


G	$=$	$(V = I \cup J, E)$	bipartite graph
b	$\in$	$\mathbb{N}^E$	bounds
c	$\in$	$\mathbb{N}^V$	interdiction costs
B	$\in$	$\mathbb{N}$	interdiction budget

$x \in \mathbb{N}^V$ is b-stable if

$$x(u) + x(v) \leq b_{uv}, \text{ for all } uv \in E.$$

Interdict $R \subseteq V$ by enforcing

$$x(v) = 0, \text{ for all } v \in R.$$

R is feasible if $c(R) \leq B$.

Formulation

1: Integer formulation

$$\begin{aligned} \text{OPT} = \min_{\substack{r\in\{0,1\}^V \\ c^T r \le B}} \max \quad & \mathbb{1}^T x \\ A^T x &\le b \\ x &\ge \mathbb{0} \\ r^T x &= 0 \end{aligned}$$

2: Interdiction to the objective

$$\begin{aligned} \text{OPT} = \min_{\substack{r\in\{0,1\}^V \\ c^T r \le B}} \max \quad & (\mathbb{1} - r)^T x \\ A^T x &\le b \\ x &\ge \mathbb{0} \end{aligned}$$

3: Dualize the max LP

$$\begin{aligned} \text{OPT} = \min \quad & b^T y \\ Ay + r &\ge \mathbb{1} \\ y &\ge \mathbb{0} \\ r &\in \{0,1\}^V \\ c^T r &\le B \end{aligned}$$

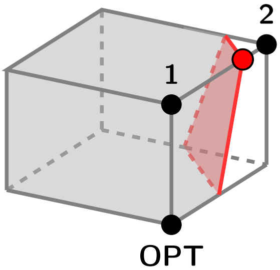
4: Relax

$$\begin{aligned} \text{OPT} \ge \min b^T y & \\ Ay + r &\ge \mathbb{1} \\ y &\ge \mathbb{0} \\ r &\ge \mathbb{0} \\ c^T r &\le B \end{aligned}$$

Sufficient conditions recap

- Feasible set is down-closed
- Objective is $\max 1^T x$
- Integral LP description is box-Totally Dual Integral

Pseudoapproximation



$$
\begin{aligned}
\text{OPT} \geq M = \quad & \min b^T y \\
& Ay + r \geq \mathbb{1} \\
& c^T r \leq B \\
& y, r \geq \mathbb{0}
\end{aligned}
$$

Pseudoapproximation guarantee

For one of $i = 1$ or $i = 2$

$$b^T y^i \leq 2M \quad \text{and} \quad c^T r^i \leq B$$

OR

$$b^T y^i \leq M \quad \text{and} \quad c^T r^i \leq 2B$$

Exploiting dual adjacency

PTAS for bipartite b-stable set interdiction

Edge Cover adjacency

$$\min b^T y + \mathbb{0}^T r + 0 r_{LR}$$

$$\begin{pmatrix} A & \mathrm{I} & 0 \\ 0 & \chi^I & 1 \\ 0 & \chi^J & 1 \end{pmatrix} \begin{pmatrix} y \\ r \\ r_{LR} \end{pmatrix} \geq \mathbb{1}$$

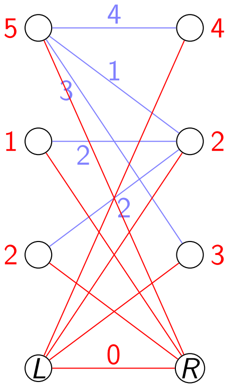
$$\mathbb{0}^T y + c^T r + 0 r_{LR} \leq B$$

$$y, r, r_{LR} \geq 0$$

Augmented graph

$G' = (V \cup \{L, R\}, E')$ with

$\Leftarrow$ incidence matrix



Theorem (Hurkens)

If edge covers $F_1, F_2 \subseteq E'$ are adj., then $F_1 \triangle F_2$ is an alternating path or cycle.

Exploiting Edge Cover adjacency for a PTAS

Observation 1

$c(\mathbf{F_2}) \leq B + 2\max_v c(v)$

Observation 2

$b(F_1) \leq \text{OPT} + 2\max_e b(e)$

If $2\max_e b(e) \leq \epsilon\text{OPT}$, then F_1 is a $(1+\epsilon)$-approximation.

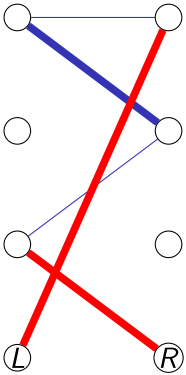
If not, guess the $\lceil \frac{2}{\epsilon} \rceil$ edges in the solution with largest b-value.

Result

$(1+\epsilon)$-approximation!

$F_1 \triangle \mathbf{F_2}$

F_1: under budget, sub-optimal
$\mathbf{F_2}$: over budget, super-optimal



Interdiction summary

2-approximation or super-optimal with cost $\leq 2B$

- Objective is maximization
- Model interdiction in the objective (down-closed)
- LP formulation with a dual integrality property & {0,1}-objective

Works for: matroid intersection, network flow, indep. systems with TU LP

improvements for some problems:

b-stable set $(1+\epsilon)$-approximation

weighted rank matroids, intersection of two matroids

weighted rank matroids with submodular costs

1-stable set interdiction in polynomial time

max stable set $G[V \setminus R^*] = |V| - |R^*| -$ max matching $G[V \setminus R^*]$